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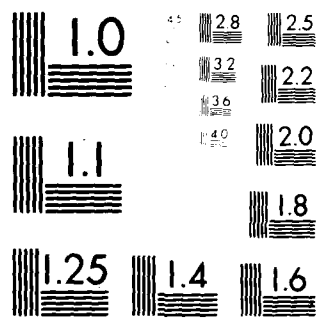
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6 FATIGUE FAILURE CRITERIA FOR UNIDIRECTIONAL FIBER COMPOSITES.

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# FATIGUE FAILURE CRITERIA FOR UNIDIRECTIONAL FIBER COMPOSITES

by

Zvi Hashin\*

## ABSTRACT

Three dimensional fatigue failure criteria for unidirectional fiber composites under states of cyclic stress are established in terms of quadratic stress polynomials which are expressed in terms of the transversely isotropic invariants of the cyclic stress. Two distinct fatigue failure modes, fiber mode and matrix mode, are modelled separately. Material information needed for the failure criteria are the S - N curves for single stress components. A preliminary approach to incorporate scatter into the failure criteria is presented.

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## 1. INTRODUCTION

A fundamental problem concerning the engineering uses of fiber composites is the determination of their resistance to combined states of cyclic stress. This problem is of much greater importance for fiber composites than for metals. For the latter there are many applications where the predominant state of stress is one dimensional. Then the fatigue resistance for constant amplitude cycling is defined by the S - N curve for this stress component.

By contrast, even in the simplest fiber composite laminate the unidirectionally reinforced laminae are in states of combined stress. In many applications it is sufficient to deal with plane stress while in others, however, a full three-dimensional state of stress must be considered. The latter case arises predominantly near holes in laminates and at laminate edges. These are complicated cases which require stress analysis by finite element methods. Analysis of fatigue failure based on the stresses obtained is not possible without failure criteria for three dimensional cyclic stress.

In view of the complexity of micro-structural damage accumulation during fatigue cycling there is little hope for resolving such problems by micromechanics methods, even for the simpler case of cycling by one stress component only. It therefore appears that at the present time the only realistic approach to such problems is to construct fatigue failure criteria for combined cyclic stress in terms of fatigue failure criteria for simple states of stress, e. g. the S - N curves for single cyclic stress components.

In this sense the problem has first been considered in [1] but only for the case of cyclic plane stress with tensile normal stresses.

Fatigue failure under combined stress has been extensively investigated in the context of metal fatigue. Early work was mostly based on such simple assumptions that principal normal stress, principal shear stress, strain energy density or distortional strain energy density associated with the cyclic stresses determine the fatigue failure. A more rational approach to the problem, of curve fitting nature, was introduced in [2]. For review of these and other approaches see e.g. [3,4]. A systematic approach to the problem has recently been given in [5]. This was based on isotropy of the material which is a reasonable assumption for metals. Consequently, the failure criteria were developed in terms of isotropic invariants of the stress tensor. The present case is much more complicated because of the pronounced anisotropy of fiber composites. However, a related approach is still feasible based on the transverse isotropy of the unidirectional material and recognition of its different failure modes. This approach is closely related to a recent method of establishment of static failure criteria, [6].

## 2. GENERAL CONSIDERATIONS

The present discussion is concerned with some general characteristics of phenomenological failure criteria for combined cyclic stress. It is assumed that the average cyclic stress  $\sigma_{ij}$  is statistically homogeneous. Averages are defined over representative volume elements (RVE) and the statistical homogeneity

implies that averages are independent of RVE location. It is further assumed that all stresses cycle at same frequency and that the maximum and minimum amplitudes, denoted  $\sigma_{ij}^2$  and  $\sigma_{ij}^1$  respectively, of each stress component remain constant during the cycling, fig. 1. The mean stresses  $\sigma_{ij}^m$  and alternating stresses  $\sigma_{ij}^a$  are defined by

$$\sigma_{ij}^m = \frac{1}{2} (\sigma_{ij}^2 + \sigma_{ij}^1) \quad (2.1)$$

$$\sigma_{ij}^a = \frac{1}{2} (\sigma_{ij}^2 - \sigma_{ij}^1)$$

If the stress cycles are sinusoidal then the cyclic state of stress is uniquely defined by the magnitudes of (2.1) and the phase lags  $\delta_{ij}$  of the individual stress components. It is thus assumed that for any given specimen the number of cycles to failure  $N$  is determined by the stress amplitudes and the phase lags. In general, therefore, the failure condition is

$$F(\sigma_{ij}^m, \sigma_{ij}^a, \delta_{ij}, N) = 1 \quad (2.2)$$

It may be noted in passing that the problem may be further complicated by allowing stress cycle amplitudes to change during cycling. An important special case is application of one stress cycle component first and then another. This situation would require cumulative damage theory under combined stress and will not be considered here.

Phase lags are encountered when the stresses are due to a number of cyclic forces which are not in phase. The writer is not aware of consideration in the literature of their influence on failure under combined stress. It will be henceforth assumed



that all stresses cycle in phase and thus the  $\delta_{ij}$  vanish. Then (2.2) becomes

$$F(\sigma_{ij}^m, \sigma_{ij}^a, N) = 1 \quad (2.3)$$

It should be noted that sign reversal of any  $\sigma_{ij}^a$  produces a cycling which is half a cycle out of phase with respect to other cycling. Thus this specific out of phase cycling is included in the formulation.

Two important special cases are vanishing alternating stress and vanishing mean stress. In the first case

$$F(\sigma_{ij}^m, 0, N) = F(\sigma_{ij}^m) = 1 \quad (2.4)$$

which is the static failure criterion, while in the second

$$F(0, \sigma_{ij}^a, N) = 1 \quad (2.5)$$

which is a failure criterion for reversed cycling.

The situation is now further simplified by assuming that the ratio

$$R_{ij} = \frac{\sigma_{ij}^1}{\sigma_{ij}^2} \quad (2.6)$$

has the same value  $R$  for all stress component cycles. This is the case when the body remains elastic under cycling since then all stresses vary linearly with the instantaneous values of the applied cyclic forces. In view of (2.1) and (2.6), (2.3) can be written as

$$F\left[\frac{1}{2} \sigma_{ij}^2 (1+R), \frac{1}{2} \sigma_{ij}^2 (1-R), N\right] = 1 \quad (2.7)$$

which will from now on be written

$$F(\sigma_{ij}, R, N) = 1 \quad (2.8)$$

where it is understood that  $\sigma_{ij}$  represent the maximum amplitudes of stress cycles, and that (2.8) is independent of the value of  $R$ . Therefore, any single component S-N curve to be used in obtaining information about the failure criterion must have this same  $R$ .

It is useful to note that by the definition (2.6) different cases of cyclic stress are defined by numerical ranges of  $R$  as follows:

$R < 0$	tension - compression
$0 \leq R < 1$	tension - tension
$1 < R$	compression - compression
$R = -1$	reversed cycling
$R = 1$	static stress

It appears that all previous work on the subject is based on the form (2.8) thus tacitly incorporating the assumptions which have been pointed out above. Consideration of failure criteria of type (2.3) would introduce tremendous additional complexity.

It is conceptually helpful to adopt the usual stress space representation of failure criteria. In such a description (2.8) for a fixed  $N$  is a surface in six dimensional stress space (or in three dimensional principal stress space). The surface is the locus of all cyclic stress states (with same frequency and same  $R$  ratio) which produce failure after  $N$  cycles. When  $N$  varies (2.8) becomes a parametric family of surfaces with parameter  $N$ , fig. 2. The static failure criterion is defined by the surface  $N=0$ . At the other extremity are states of cyclic stress which will not produce failure for any practically attainable,

theoretically infinite, number of cycles. These define a region in stress space which contains no failure points and may be called the fatigue limit region, fig. 2.

A state of stress  $\sigma_{ij}$  which produces failure after  $N$  cycles may be regarded as a vector in stress space connecting the origin to the appropriate point on the  $N$  failure surface. It is to be expected on physical grounds that if all  $\sigma_{ij}$  are increased in fixed mutual ratios the number of cycles to failure will decrease. It follows that the failure surface for  $N_2$  is contained within the failure surface for  $N_1 < N_2$ . Consequently (2.8) is a non-intersecting family of surfaces which are all contained within the static failure surface, fig. 2. (It should be noted that the foregoing reasoning disregards scatter.)

Since infinite failure stresses do not occur in nature the failure surfaces should be closed. However, an infinite failure stress may at times be a convenient mathematical idealization to express the fact that a failure stress for one situation is larger by an order of magnitude than for another. For example: failure under hydrostatic compression as compared to failure under uniaxial stress. A well known case is the Mises representation of a plasticity yield surface as a cylinder which extends to infinity in octahedral direction.

### 3. FAILURE CRITERIA

For the purpose of establishment of failure criteria under combined cyclic stress it is essential to recognize the different fatigue failure modes of unidirectional fiber composites.

The most common fiber composite consists of polymeric matrix reinforced by Glass, Graphite, Carbon or Boron fibers. When a

specimen of such a composite is subjected to tension - tension cycling in fiber direction it is mostly observed that damage accumulates randomly in the form of many small cracks in fiber direction (axial) and normal to the fibers (transverse). The axial cracks are in the matrix while the transverse cracks rupture fibers and matrix. It is the accumulation of the latter which reduces the carrying capacity of the specimen resulting finally in specimen failure with a jagged irregular failure surface, fig. 3. This kind of failure mode is very different from a metal fatigue failure mode which consists of the initiation and propagation of a single crack.

No doubt transverse crack propagation in a fiber composite is inhibited by the fibers since they are much stiffer and stronger than the matrix. But it must be borne in mind that the transverse properties of the fibers are the ones which are important here. Glass and Boron fibers are reasonably isotropic, thus have high transverse stiffness. By contrast carbon and graphite are very anisotropic and have low transverse stiffness - at times as low as twice the polymeric matrix stiffness. Indeed, the damage accumulation mode is mostly observed in the case of Glass and Boron fibers while for Carbon and Graphite a sudden transverse crack propagation mode also occurs. The common feature of the two modes is that failure takes place by fiber rupture through transverse cracks. Thus this type of failure will be called fiber mode.

Consider a homogeneous anisotropic elastic brittle specimen which has the elastic symmetry of a unidirectional fiber composite,

thus is transversely isotropic, fig. 4, and contains transverse cracks. Let the specimen be subjected to a homogeneous stress state  $\sigma_{ij}$ . Evidently, the cracks have no effect on the stress components  $\sigma_{22}$ ,  $\sigma_{33}$  and  $\sigma_{23}$ . Therefore crack criticality depends only on the stress components  $\sigma_{11}$ ,  $\sigma_{12}$  and  $\sigma_{13}$ . It is an open question whether cracks in a fiber composite obey the laws of anisotropic fracture mechanics, but it may be expected that the situation with respect to the effect of the average components  $\sigma_{22}$ ,  $\sigma_{33}$  and  $\sigma_{23}$  will be similar. It will consequently be assumed that the fiber failure mode depends only on the average stress components  $\sigma_{11}$ ,  $\sigma_{12}$  and  $\sigma_{13}$ .

When the cycling is other than tension-tension the situation is less clear. It may be reasoned that for tension-compression cycling the preceding arguments are still acceptable because of the presence of the tension component and thus the assumption of independence of failure of  $\sigma_{22}$ ,  $\sigma_{33}$  and  $\sigma_{23}$  can be retained.

The failure mechanisms in compression-compression cycling are much less understood than for tension-tension. There is sufficient evidence that in static compression in fiber direction the composite fails because of fiber buckling. Approximate stability analyses [7,8] have indicated that the compressive strength is proportional to the matrix shear modulus. The matter of the failure mechanism in compression-compression cycling is an open question. It is possible that the fibers will buckle at cyclic load much lower than static buckling load because of deterioration of matrix shear modulus due to cycling and/or due to opening of longitudinal cracks at fiber/matrix interfaces.

In view of all of these uncertainties it is difficult to form an opinion whether or not transverse cyclic normal and shear stress will affect the longitudinal compressive cyclic failure stress.

The second primary failure mode of a unidirectional composite consists of a planar crack in fiber direction, in between the fibers, and is called the matrix mode. This occurs in static and cyclic loading and is best demonstrated with off-axis specimens (see e. g. [1]). Such cracks occur suddenly without warning and propagate at once through the specimen. This is in contrast to fatigue cracks in metals which grow slowly with the cycling.

In a thin flat specimen in plane stress the fracture surface of the matrix mode is plane perpendicular to the specimen plane. It is to be expected, that for a three-dimensional state of stress the fracture surface will be a plane in fiber direction oriented at some angle in the transverse plane, fig. 5. The stress  $\sigma_{11}$  has no effect on the propagation of such a crack and it is therefore assumed that it does not enter into the matrix failure criterion.

There arises again the question whether the failure mode described also occurs for tension-compression and compression-compression fatigue. Pending thorough experimental investigation of this question it seems reasonable to expect that the directionality of fibers will lead also in these cases to the same kind of plane matrix failure mode.

Assuming that all stress components cycle in phase with same R ratios the failure criteria are of the form (2,8). In view of

the above discussion of failure modes the fiber and matrix failure criteria have the forms

$$F_f(\sigma_{11}, \sigma_{12}, \sigma_{13}, R, N) = 1 \quad (a) \quad (3.1)$$

$$F_m(\sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{13}, R, N) = 1 \quad (b)$$

where subscripts f and m indicate fiber and matrix modes, respectively, and all stresses are maximum amplitudes. It should be recalled that the reasoning leading to (3.1a) does not necessarily apply to compression-compression fatigue.

A unidirectional fiber composite is initially transversely isotropic in the macrosense with respect to the  $x_1$  axis in fiber direction. It is not unreasonable to assume that the material will retain this macro-symmetry during the cycling until failure occurs. It follows that the failure criteria must be functions of the transversely isotropic invariants of the cyclic stress tensor and thus can be developed by a method similar to one employed in [6]. The invariants are

$$I_1 = \sigma_{11} \quad (a) \quad (3.2)$$

$$I_2 = \sigma_{22} + \sigma_{33} \quad (b)$$

$$I_3 = \sigma_{23}^2 - \sigma_{22}\sigma_{33} \quad \text{or} \quad \frac{1}{4} (\sigma_{22} - \sigma_{33})^2 + \sigma_{23}^2 \quad (c)$$

$$I_4 = \sigma_{12}^2 + \sigma_{13}^2 \quad (d)$$

$$I_5 = 2 \sigma_{12}\sigma_{23}\sigma_{13} - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2 \quad (e)$$

The second of (c) and (d) are squares of principal shear stresses in transverse and axial planes respectively. It follows that (3.1) assume the forms

$$F_f(I_1, I_4, R, N) = 1 \quad (3.3)$$

$$F_m(I_2, I_3, I_4, I_5, R, N) = 1$$

Assuming failure criteria which are quadratic in the stresses (see [6] for discussion of this common approximation) (3.3) reduce to

$$A_f I_1 + B_f I_1^2 + D_f I_4 = 1 \quad (3.4)$$

$$A_m I_2 + B_m I_2^2 + C_m I_3 + D_m I_4 = 1$$

where the coefficients are functions of  $R$  and  $N$ .

Specializing (3.4) to the cases of pure transverse and axial cyclic shears,  $\sigma_{23}$  and  $\sigma_{12}$  (or  $\sigma_{13}$ ), respectively, it follows at once that

$$C_m = \frac{1}{\tau_T^2} \quad (3.5)$$

$$D_m = D_f = \frac{1}{\tau_A^2}$$

where

$$\tau_T = \tau_T(R, N) \quad (3.6)$$

$$\tau_A = \tau_A(R, N)$$

are the fatigue shear failure stresses as represented by the S-N curves for these stresses. Similarly, let the S-N curves for cyclic stress  $\sigma_{11}$  in fiber direction and  $\sigma_{22}$  or  $\sigma_{33}$  transverse to fibers be given by



$$\sigma_A = \sigma_A (R, N) \quad (3.7)$$

$$\sigma_T = \sigma_T (R, N)$$

It follows from (3.4) and (3.2) that

$$A_f \sigma_A + B_f \sigma_A^2 = 1 \quad (3.8)$$

$$A_m \sigma_T + B_m \sigma_T^2 = 1$$

Since this is insufficient information to determine the coefficients it is in general necessary to perform combined cyclic stress failure tests e. g. cyclic  $\sigma_{11}$ ,  $\sigma_{12}$  or cyclic  $\sigma_{22}$ ,  $\sigma_{12}$  to obtain necessary additional equations from (3.4). Such tests can be carried out by combined torsion-axial force cycling of thin walled cylinders in which the fibers are either in axial or in circumferential direction, fig. 6. Such tests are, however, quite expensive because of the high cost of the specimens.

It will now be shown that in the case of fully reversed cycling, i. e. mean stresses vanish, thus  $R = -1$  for all stress cycles, all of the coefficients in (3.8) are easily determined. The reason is that a change of sign of reversed cyclic stress cannot affect the lifetime (number of cycles to failure) for it merely displaces all cycles by half a period. Consequently (3.8) must be valid with same coefficients for  $\sigma_A$ ,  $-\sigma_A$  and  $\sigma_T$ ,  $-\sigma_T$ . It follows that

$$A_f = A_m = 0 \quad (3.9)$$

$$B_f = \frac{1}{\sigma_A^2}$$

$$B_m = \frac{1}{\sigma_T^2}$$

Consequently the failure criteria for reversed cycling are:

fiber mode

$$\left(\frac{\sigma_{11}}{\sigma_A}\right)^2 + \frac{\sigma_{12}^2 + \sigma_{13}^2}{\tau_A^2} = 1 \quad (a) \quad (3.10)$$

matrix mode

$$\left(\frac{\sigma_{22} + \sigma_{33}}{\sigma_T}\right)^2 + \frac{\sigma_{23}^2 - \sigma_{22}\sigma_{33}}{\tau_T^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{\tau_A^2} = 1 \quad (b)$$

where

$$\sigma_A = \sigma_A(-1, N) \quad \sigma_T = \sigma_T(-1, N) \quad (3.11)$$

$$\tau_A = \tau_A(-1, N) \quad \tau_T = \tau_T(-1, N)$$

are the failure stresses for reversed cycling as given by their S-N curves. For any given state of cyclic stress (3.10-11) should be regarded as equations with unknown N, the number of cycles to failure, also called - the fatigue life. Each of (3.10-11) is solved separately and failure is determined by the criterion which leads to the smaller N. This procedure determines fatigue life and failure mode.

In view of the simplicity of (3.10-11) it is tempting to use them also for the more general case when the mean cyclic

stresses do not vanish, in which case the cyclic failure stresses are (3.6-7). It is possible to advance some fracture mechanics arguments that such a presentation would be appropriate for tension-tension cycling, but only testing data under combined stress can form a basis for examination of such an assumption. Given all the uncertainties and the significant scatter of fatigue data the writer believes that the failure criteria (3.10-11) can probably be used on an ad-hoc basis for non-reversed cycling. Some evidence for this is known for plane stress and will be shown further below.

An essential problem in the definition of the failure criteria is the experimental determination of the failure stresses (3.6-7) or (3.11). The simplest is  $\sigma_A$  which is obtained from the S-N curve for stress in fiber direction, using flat unidirectional specimens. The  $\sigma_T$  S-N curve can in principle be obtained with similar specimens in which the fibers are in transverse ( $90^\circ$ ) direction but experience shows that such specimens are not suitable because of the very large scatter of the test data. To obtain the  $\tau_A$  S-N curve directly it is necessary to employ thin walled tubes in torsion. Since fatigue testing requires many specimens this is a very expensive procedure. A better alternative is to obtain the  $\sigma_T$  and  $\tau_A$  S-N curves by use of off-axis specimens. This will be discussed further below.

Experimental determination of  $\tau_T$  is a notoriously difficult problem even for static loading. The primary difficulties are that torsion cannot be used and that flat specimens must be made

with fibers normal to their planes. Such specimens can only be produced by transverse cuts through unidirectional specimens and therefore their sizes must be small. One possibility is to employ a new device for producing pure shear, [9], which can be used with very small specimens. In the absence of other information it is reasonable to assume that  $\tau_T$  is the matrix shear failure stress.

The basic state of stress of a unidirectional lamina within a laminate is uniform plane stress. If  $x_1$  is in fiber direction,  $x_2$  transverse to it in the plane of the lamina and  $x_3$  normal to the lamina then the plane components of stress are  $\sigma_{11}$ ,  $\sigma_{22}$ ,  $\sigma_{12}$  all others vanish. The failure criteria (3.10-11) reduce to:

fiber mode

$$\left(\frac{\sigma_{11}}{\sigma_A}\right)^2 + \left(\frac{\sigma_{12}}{\tau_A}\right)^2 = 1 \quad (a) \quad (3.12)$$

matrix mode

$$\left(\frac{\sigma_{22}}{\sigma_T}\right)^2 + \left(\frac{\sigma_{12}}{\tau_A}\right)^2 = 1 \quad (b)$$

Failure will occur in the mode which corresponds to lower fatigue lifetime.

Next there is considered the case of a rectangular flat specimen subjected to cyclic uniform uniaxial stress unidirectionally reinforced at direction  $\theta$  with respect to specimen axis, fig. 7. The stresses with respect to the material system of axes  $x_1$ ,  $x_2$  are

$$\sigma_{11} = \sigma \cos^2 \theta \quad (3.13)$$

$$\sigma_{22} = \sigma \sin^2 \theta$$

$$\sigma_{12} = \sigma \cos \theta \sin \theta$$

Insertion of (3.13) into (3.12) leads to

$$\left(\frac{\sigma}{\sigma_A}\right)^2 \cos^4 \theta + \left(\frac{\sigma}{\tau_A}\right)^2 \cos^2 \theta \sin^2 \theta = 1 \quad (a) \quad (3.14)$$

$$\left(\frac{\sigma}{\sigma_T}\right)^2 \sin^4 \theta + \left(\frac{\sigma}{\tau_A}\right)^2 \cos^2 \theta \sin^2 \theta = 1 \quad (b)$$

When an off-axis specimen is cycled to failure for different stress levels the test data can be summarized by the S-N curve  $\sigma(R,N)$ . Eqs. (3.14) may be regarded as relations among  $\sigma(R,N)$ ,  $\sigma_A(R,N)$ ,  $\sigma_T(R,N)$  and  $\tau_T(R,N)$ . It is easily realized that the fiber failure mode (3.14a) will apply for angles  $0 \leq \theta \leq \theta_0$  while the matrix mode (3.14b) will apply for angles  $\theta_0 \leq \theta \leq 90^\circ$ . The transition angle  $\theta_0$  is defined by simultaneous satisfaction of the failure criteria (3.14). This defines  $\theta_0$  as

$$\tan \theta_0 = \sqrt{\frac{\sigma_T}{\sigma_A}} \quad (3.15)$$

This angle is generally small for polymer matrix composites since  $\sigma_T \ll \sigma_A$ .

In order to find the S-N curves  $\sigma_T(R,N)$  and  $\tau_A(R,N)$  it is best to fail the specimen in matrix modes using specimens with several different  $\theta$ . Using two convenient values  $\theta_1, \theta_2$  - (3.14b) become

$$\frac{\sin^4 \theta_1}{\sigma_T^2(R,N)} + \frac{\cos^2 \theta_1 \sin^2 \theta_1}{\tau_A^2(R,N)} = \frac{1}{\sigma^2(R,N)} \quad (3.16)$$

$$\frac{\sin^4 \theta_2}{\sigma_T^2(R,N)} + \frac{\cos^2 \theta_2 \sin^2 \theta_2}{\tau_A^2(R,N)} = \frac{1}{\sigma^2(R,N)}$$

which can be solved for  $\sigma_T$  and  $\tau_A$ .

Such a procedure has been successfully used, [1], to represent unidirectional glass/epoxy test data. In this work the simple failure criterion  $\sigma_{11} = \sigma_A$  was used instead of (3.12a). Figs. 8-9, taken from [1], show the fit of the failure criteria to the experimental data for fatigue failure tests of glass/epoxy off axis specimens with various angles of reinforcement. It is seen that there is reasonably good agreement with the data although (3.12) are applicable to reversed cycling and not necessarily for the present case which includes mean stress;  $R = 0.1$ .

#### 4. THE PROBLEM OF SCATTER

An intrinsic characteristic of fatigue test data is large scatter. It is not unusual to observe a decade of scatter for fatigue lives associated with same cyclic stress. The role of a deterministic failure criterion vis-a-vis such scattered data is not clear and does not appear to have been explored. It is customary to draw the failure locus on a plot of the test data and to be satisfied when the locus passes "through" the data. A more rational point of view may be proposed: given the scattered data for simple tests such as expressed by the S-N curves (3.6-7). To predict statistical aspects such as means and

variances for failure under combined states of cyclic stress. A possible approach to this problem is given in the following.

It is reasonable to assume that the scatter in lifetimes of specimens which are subjected to identical cyclic stress states is due to the differences in microstructures and consequent different evolution of microfailures in test specimens. Conversely, it is reasonable to assume that if a specimen could be reproduced exactly, in all microdetails, in any number of replicas, the resulting specimens would exhibit no lifetime scatter whatsoever. In what follows such hypothetical specimens will be called clones and clones of a certain kind will be called a clone species.

Evidently a clone species obeys some deterministic failure criterion which is unknown and can unfortunately not be experimentally investigated since clone specimens are not available. It is however possible to relate any proposed deterministic failure criterion to scattered experimental data in indirect fashion. Suppose that a group of specimens has been tested to failure under identical cyclic stress states. Each specimen is regarded as a member of a hypothetical clone species. It therefore obeys a deterministic failure criterion which results in an expression for failure prediction, involving certain parameters (e.g. one dimensional cyclic failure stresses (3.6-7)). Since however, such parameters are different for different species, the failure criterion predicts a different lifetime and different states of failure stress for each specimen i.e. for its clone species. Therefore the predicted failure stress statistics is

determined by the specimen parameter statistics in terms of the analytical expressions or procedures resulting from the failure criterion proposed. Consequently, experimental tests of a failure criterion should consist of comparisons of such quantities as probabilities, means and variances of failure stress components and lifetimes as predicted by the theory and as determined by experiment.

This approach will be illustrated by a simple example. Suppose that the state of stress is plane and the specimens all fail in the matrix mode obeying the failure criterion (3.12b). Consider stress states for which the ratio  $\sigma_{12}/\sigma_{22}$  is constant. Thus

$$\sigma_{22} = \sigma s_{22} \quad (4.1)$$

$$\sigma_{12} = \sigma s_{12}$$

where  $s_{22}$  and  $s_{12}$  are given non dimensional numbers and  $\sigma$  has dimensions of stress. Let the  $m^{\text{th}}$  specimen have cyclic failure stresses  $\sigma_{Tm}(R,N)$  and  $\tau_{Am}(R,N)$ . Then from (4.1) the failure stress  $\sigma_m(R,N)$  for this specimen is given by

$$\sigma_m(R,N) = \left[ \left( \frac{s_{22}}{\sigma_{Tm}} \right)^2 + \left( \frac{s_{12}}{\tau_{Am}} \right)^2 \right]^{-1/2} \quad (4.2)$$

which defines the failure stresses (4.1) for this specimen and for its clone species.

If failure tests with cyclic stress (4.1) are conducted with  $M$  specimens then the mean of  $\sigma$  is given by



$$\langle \sigma \rangle (R, N) = \frac{1}{M} \sum_{m=1}^M \sigma_m [\sigma_{Tm}(R, N), \tau_{Am}(R, N)] \quad (4.3)$$

and the variance  $v$  is

$$v (R, N) = \frac{1}{M} \sum_{m=1}^M [\sigma_m - \langle \sigma \rangle]^2 \quad (4.4)$$

The mean (4.3) defines through (4.1) the mean failure stresses

$$\langle \sigma_{22} \rangle = \langle \sigma \rangle s_{22} \quad (4.5)$$

$$\langle \sigma_{12} \rangle = \langle \sigma \rangle s_{12}$$

If this procedure is carried out for different values of  $s_{22}$  and  $s_{12}$  there is obtained a mean failure locus defined by the stresses (4.5) and the associated variances are given by (4.4).

Often static failure criteria such as (3.14) are used with scattered data by substituting for  $\sigma_A$ ,  $\sigma_T$  and  $\tau_A$  their means obtained by one dimensional failure experiments. It should be emphasized that the procedure outlined above is entirely different from such an arbitrary approach. Only when there is little scatter, thus small variance, can these two different approaches be expected to give similar results.

It must now be pointed out that the present procedure for obtaining mean failure stresses and variances for combined cyclic stress involves a serious practical difficulty. It is seen that the averaging in (4.3) is carried out for constant  $N$ . However, fatigue tests are carried out for constant stress with the scatter in associated  $N$  since stress can be controlled by a testing

machine but  $N$  cannot. To carry out the required "vertical" stress average, fig. 10, it is necessary either to have a very large number of test data or to devise the joint probability function for  $\sigma$  and  $N$ . It should, however, be emphasized that the present method is equally valid for static failure criteria where that difficulty does not arise.

The problem of obtaining statistical information for the scattered  $N$  associated with given state of cyclic stress also involves serious difficulties. In order to average  $N$  (4.2) must first be solved for  $N$  but this is not possible since the functional dependence of  $\sigma_m$ ,  $\sigma_{Tm}$  and  $\tau_{Am}$  on  $N$  is not known for a clone species.

## 5. CONCLUSIONS

The problem of establishment of fatigue failure criteria for unidirectional fiber composite subjected to three dimensional cyclic stress has been considered in terms of quadratic approximations and on the basis of the transverse isotropy of the material. A fundamental ingredient of the approach is the identification of different fatigue failure modes - fiber and matrix modes, and their individual modeling.

A special interesting situation is reversed cycling for in this case the one dimensional cycle failure stresses  $\sigma_A$ ,  $\sigma_T$ ,  $\tau_A$  and  $\tau_T$  as defined by their S-N curves are sufficient information to determine the failure criteria within the frame of the quadratic approximation. For cyclic stresses with any R ratio some testing information for combined stress cycling is needed. It may be that the simple failure criteria for reversed cycling are suitable approximations for this more general case. Some experimental

evidence to this effect, for plane stress cycling, has been presented.

Fatigue failure criteria for three dimensional cyclic stress are of particular importance for fatigue failure analysis of notched laminates, an important aeronautical engineering problem. The states of stress in the laminae near the notch are three dimensional and can only be obtained by numerical methods such as finite element methods. The failure criteria can be used to predict after how many cycles and at which location first failure occurs and in which mode. Location means a specific finite element. If the element fails in the fiber mode then it may be assumed that its axial (in fiber direction) Young's modulus is zero. If it fails in the matrix mode, the transverse Young's modulus and the shear stiffness may be assumed negligible. The numerical analysis can now be continued with the new properties until more elements fail, etc. Such an approach is illustrated by a notched laminate analysis given in [10].

An important additional failure mode which is specific to laminates is delamination, that is separation of the bond between two laminae due to shear and normal stresses. This failure mode has not been considered here since the work is concerned with unidirectional materials.

Finally, the incorporation of the statistics of the failure test data is a most important aspect of fatigue failure criteria. A preliminary approach to this problem has been given here but further treatment to overcome the difficulties mentioned is needed.

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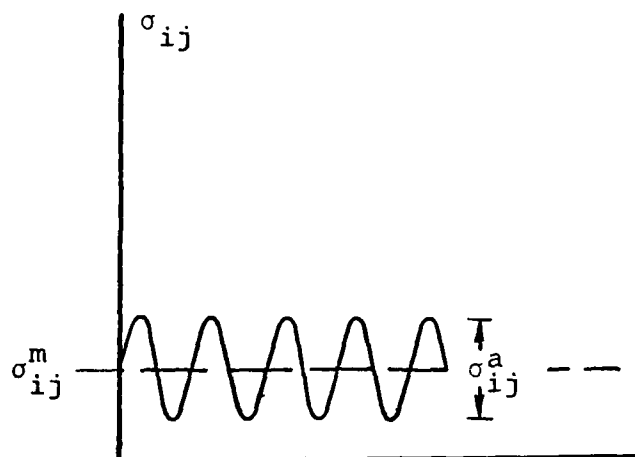


Fig. 1. Cyclic Stress

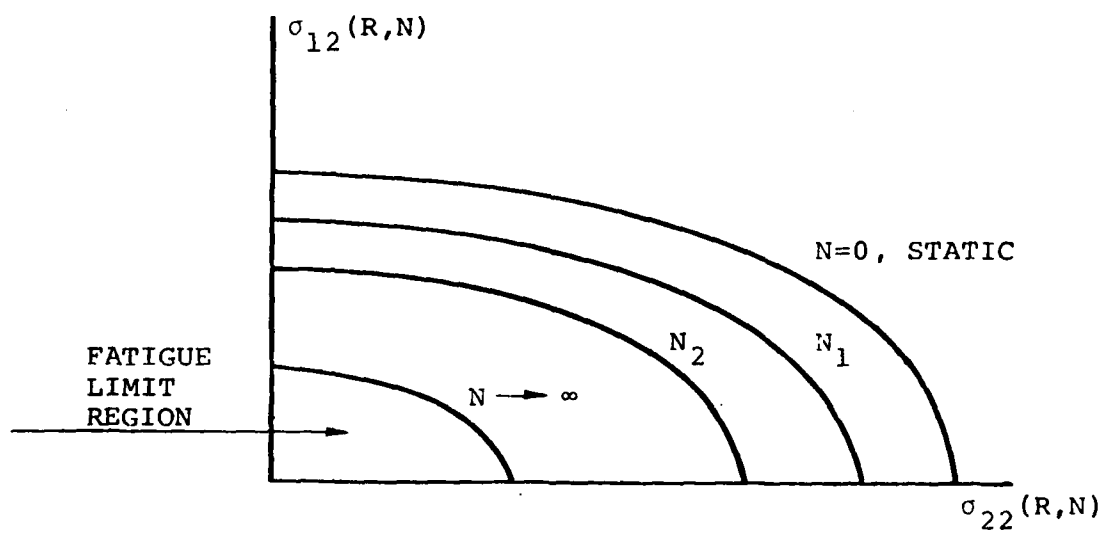
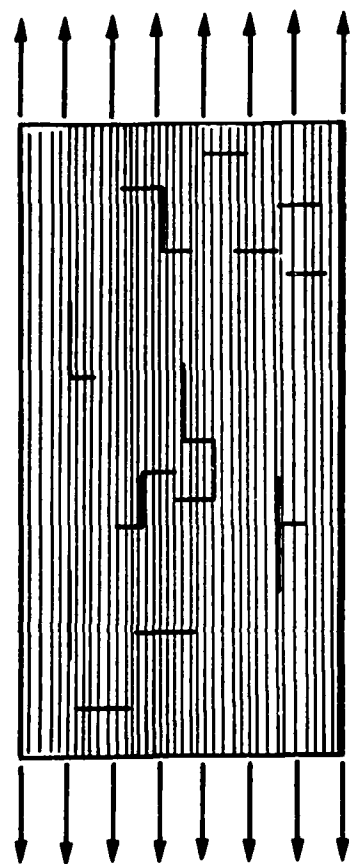
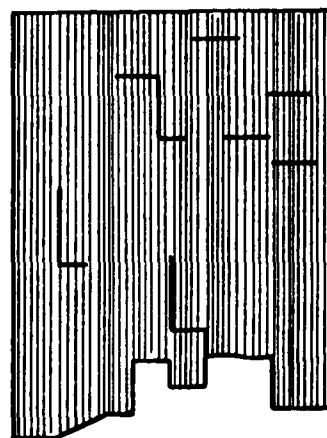


Fig. 2. Fatigue Failure Surface Family



(a)



(b)

Fig. 3. Fiber Failure Mode

- (a) Damage Accumulation
- (b) Failure

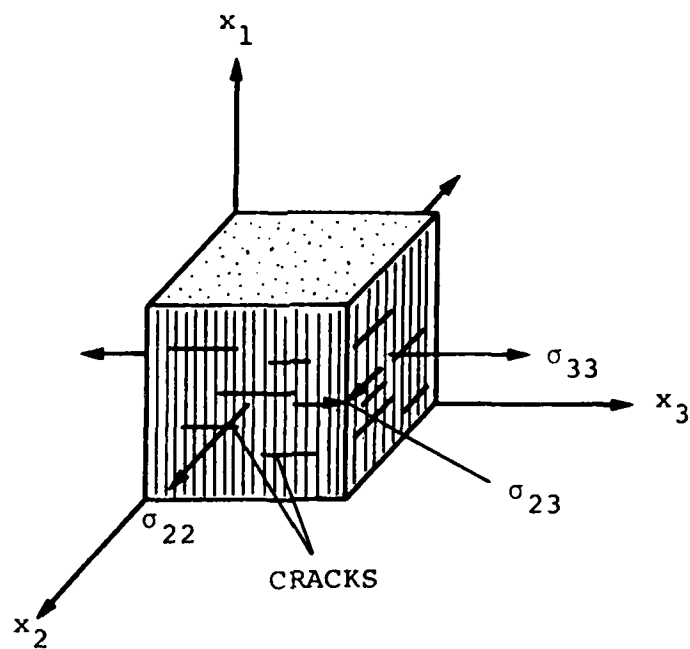


Fig. 4. Transverse plane cracks have no effect on transverse stress

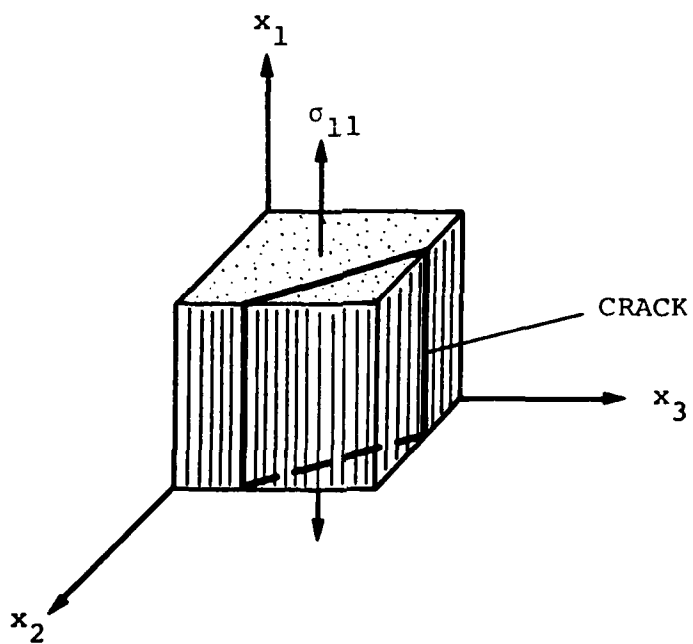


Fig. 5. Axial crack has no effect on axial stress

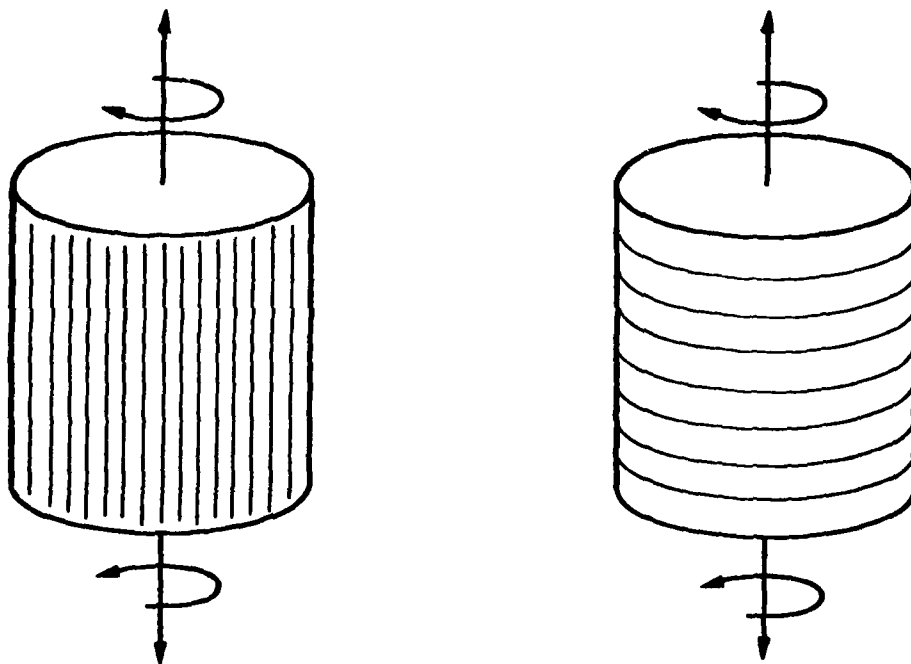


Fig. 6. Thin-Walled Torsion Specimens

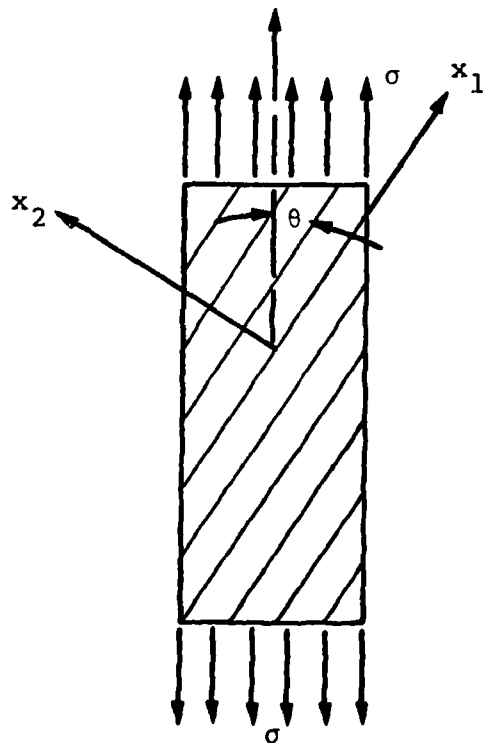
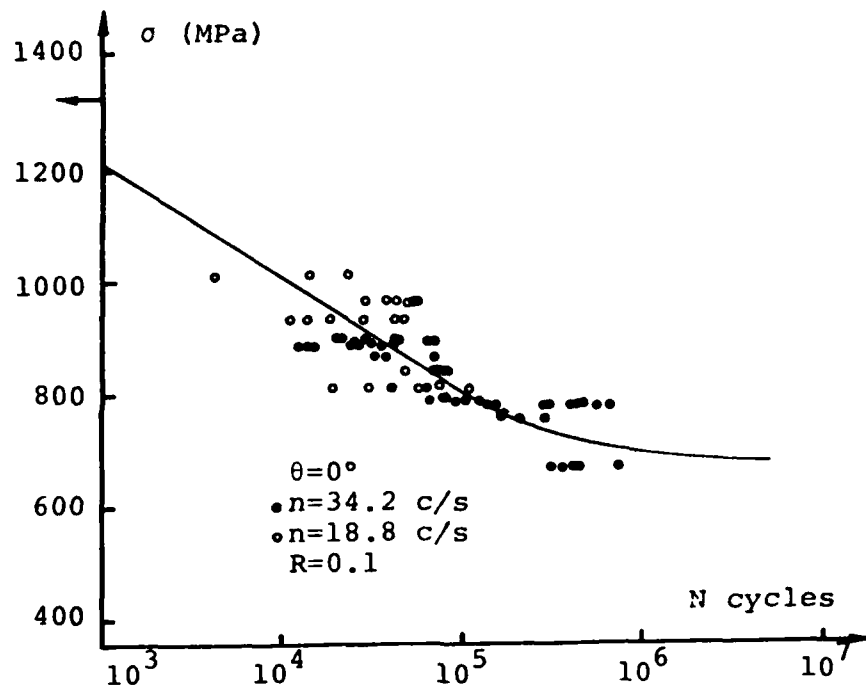


Fig. 7. Off-Axis Specimen





S-N curve for stress in fiber direction

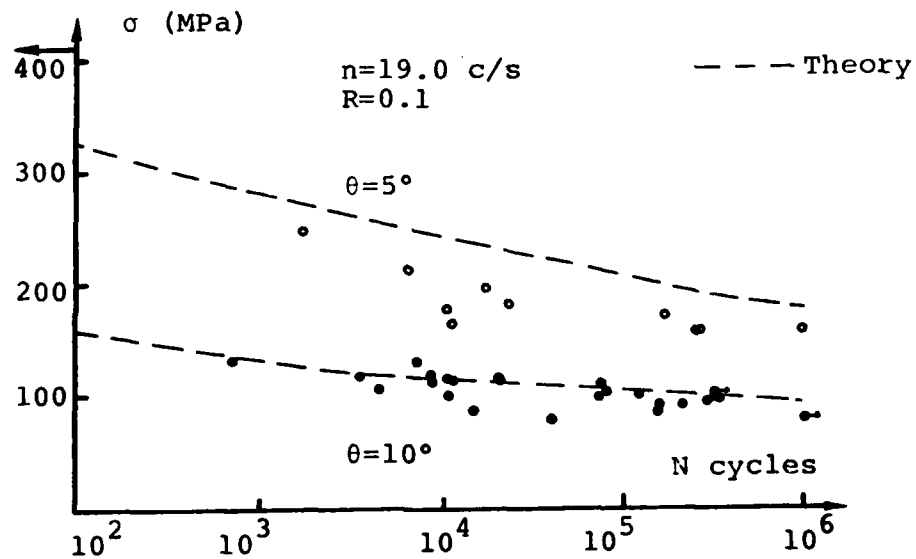


Fig. 8. Test data for off-axis specimens and theory

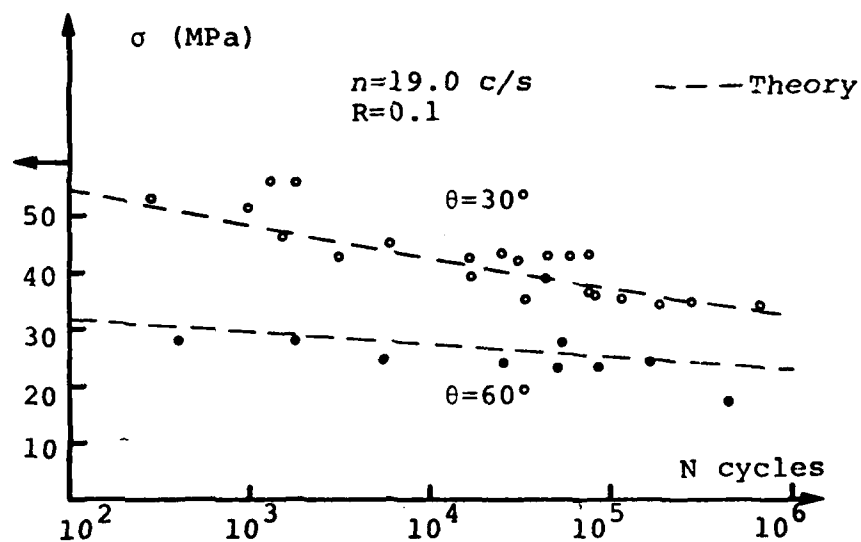
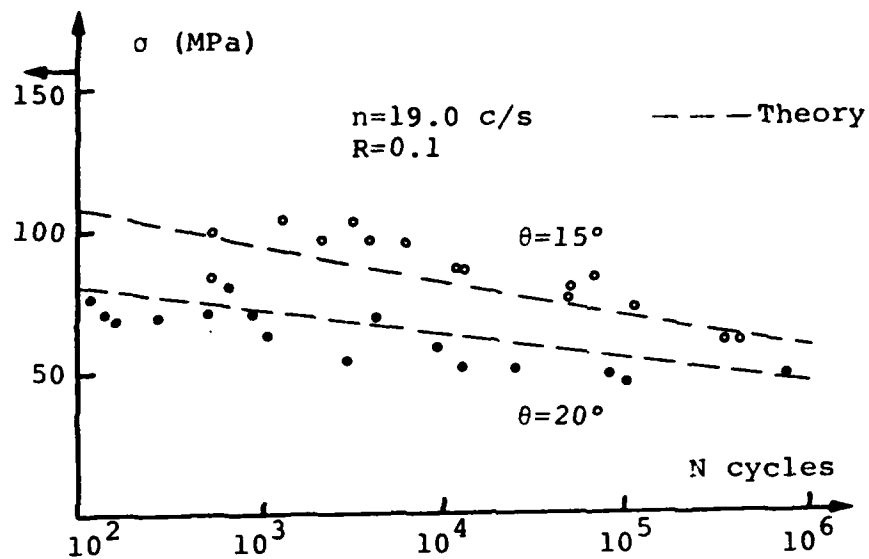


Fig. 9. Test data for off-axis specimens and theory

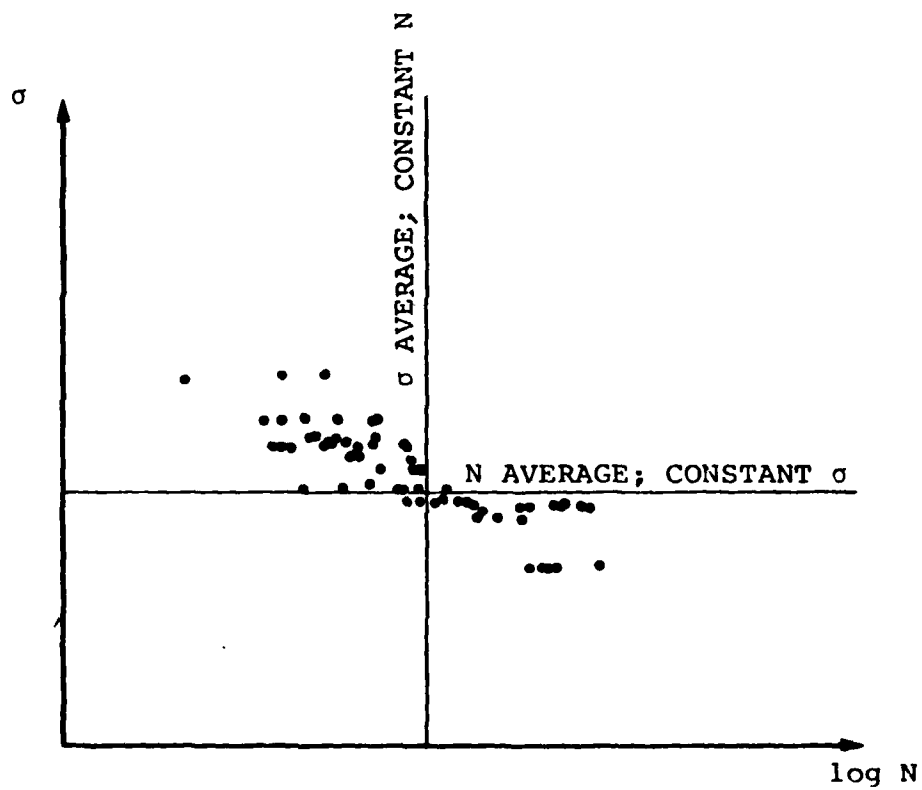


Fig. 10. Test Data Averaging